Lab 3: RC Circuits

Prelab

Deriving equations for the output voltage of the voltage dividers you constructed in lab 2 was fairly simple. Now we want to derive an equation for the output voltage of a circuit with both resistance and reactance, which will require a brief foray into the field of complex analysis.

- Watch the Lab 3 video demonstrating the derivation for the output voltage of the circuit shown in figure 2 (On moodle or `www.youtube.com/watch?v=nAsnk1Yj4u8`).

Q. Following the same procedure, derive the output voltage for the circuit shown in figure 4. **You will need this equation in lab.**

Q. For both circuits (fig 2 and fig 4) what are the phase angle and magnitude of V_{out} (relative to V_{in}) when $\omega = 1/(RC)$?

- Construct circuit 2 in EveryCircuit.
- Set values for the capacitor and resistor to match those in figure 2 and set the frequency to $1/(2\pi RC)$.
- Select the input and output voltage nodes (wires) and in each case click the "eye" icon to show oscilloscope plots. Click on the "t" to start the analysis and on the clock icon to zoom on the time axis.

Q. Export an image of the circuit to include in your lab report and be prepared to show the working simulation at the start of lab. Does the phase shift between the voltages in the simulation agree with the one you found above?

Part I: Capacitor Charging and Discharging

When charging or discharging a capacitor, the voltage difference across it has an exponential dependence on time, as shown in equations (1.1) and (1.2). The time constant τ is equal to the product of the series resistance and capacitance, ($\tau = RC$).

$$V_{out}(t) = V_{max}(1 - e^{-t/\tau}) \qquad \text{(Charging)} \qquad (1.1)$$

$$V_{out}(t) = V_{max}(e^{-t/\tau}) \qquad \text{(Discharging)} \qquad (1.2)$$

A charging capacitor ends at V_{max} and a discharging capacitor starts at V_{max}.

Q. In each case, after a period of time equal to the time constant (that is $t = \tau$), what percent of V_{max} will the capacitor voltage be at?

Q. What integer number of times constants must pass for a charging capacitor's voltage to be above 99% of the max voltage?

You are going to use your scope to measure the time constant for the circuit in figure 1 and compare it to a direct calculation.

Q. After using your multimeter to find more exact values for the resistor and capacitor you will use in circuit 1, calculate the time constant from their values.

Q. Perform a full uncertainty propagation to determine $\Delta\tau$ as a function of R, ΔR, C, and ΔC.

1. Build the circuit shown in figure 1, drive it with a 7-8 kHz square wave. (Make sure the duty cycle know is turned to the "Cal" position.)
2. Reset your scope by hitting the "DEFAULT SETUP" button
3. Be sure both the probes and each scope channel are in 10X mode
4. Connect channel 1 to view the square wave and channel 2 to view the capacitor voltage
5. To clean up the signal, set ACQUIRE to average 4 or 16 samples
6. Move the signals so that the "1" arrow and "2" arrow are both vertically centered on the screen (such that the signals are directly on top of each other)
7. Adjust the zoom and position settings until the the charging cycle nearly fills the screen
8. Set the cursor to voltage mode and find delta V between the top and bottom, this is your V_{max}. Using your calculations from above, determine what the voltage level should be after 1 time constant. Move the top cursor until the delta V between the cursors equals that value.
9. Find where the horizontal line of the top cursor intersects the capacitor voltage curve. Move the signals horizontally until that intersection point is aligned with the center axis of the grid.
10. Switch cursors to time mode and measure the amount of time from when the signal begins to rise until it reaches the voltage level you determined.
11. Repeat this procedure for the discharge cycle.

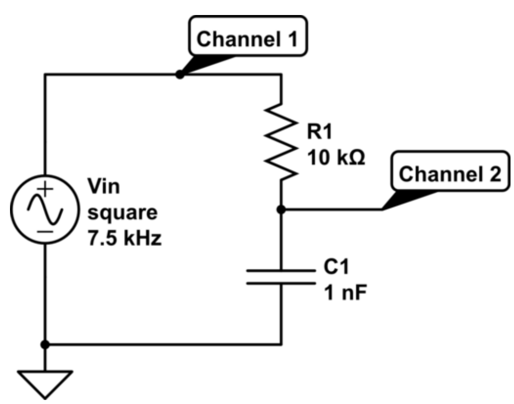

Figure 1: RC Circuit

Q. What time constant did you measure on the charge cycle? On the discharge cycle?

Q. Take a picture of a full charge and discharge cycle to include in your report.

Part II: Capacitor Voltage

Now that we've examined the step response of the RC circuit we are going to examine the response of the capacitor voltage to a sine wave.

For part II, use the equation for voltage from the end of the video.

2.1 AC analysis of an RC circuit

- Switch your function generator to output a sine wave with the same frequency as in the previous section and set the amplitude to maximum. You should see two sine waves (input and output) with a phase shift.

Q. Using the equations from the prelab, calculate what this phase shift should be (in degrees).

Q. Use the MEASURE menu to find the period of the sine wave

Q. From the period, T, and the phase shift, ϕ, calculate what you expect the time lag to be. That is, solve: $\phi/360 = \Delta t/T$ for Δt.

Q. Using the cursors, measure the time shift between the input and output waveforms and compare to the value you calculated.

2.2 RC circuit as an integrator

To achieve the greatest possible phase shift, based on the phase angle equation for the voltage across the capacitor, the inverse tangent term would need to be zero. This can't actually happen since it would require the frequency to go to infinity. But we can get close, and when the output has a phase angle close to this maximum of -90 degrees, the output is very nearly the integral of the input (multiplied by an amplitude factor).

The frequency $f_0 = 1/(2\pi RC)$ is a special frequency known as the cutoff frequency. As noted in the video, at this frequency the phase shift is 45°.

Q. As a function of f_0, what frequency will give a phase shift of -89 degrees? To get closer to $-90°$, would the frequency need to go higher or lower than that value?

Q. At this frequency, what will the amplitude of the output be (as a fraction of V_{in})? How does the amplitude depend on frequency?

- Obtain components R_1 and C_1 as shown in figure 2. Use your multimeter to obtain more accurate values for your components.
- Plug these component values into your result from above and set your function generator to output a sine wave at that frequency.
- Set the output amplitude of the function generator to maximum.
- Build the circuit shown in figure 2.
- Set each scope channel to AC coupling (this strips off DC offsets which sometimes creep into the signal)

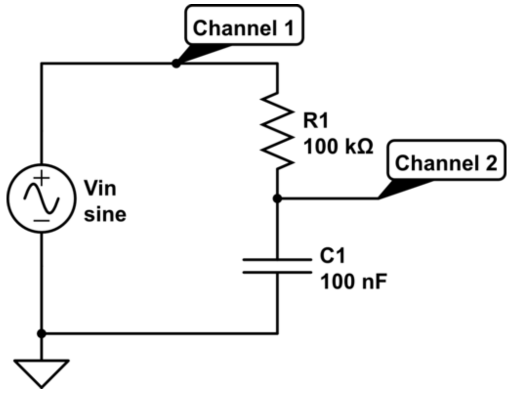

Figure 2: An RC circuit which outputs the integral of the input.

If you did everything correctly, the voltage waveforms should have a phase difference of 89 degrees with the output lagging the input.

Q. Measure the Δt between the voltage waveforms with the cursors, and the period with the MEASURE menu. Convert Δt to degrees. Did you get a reasonable result?

Q. Measure the peak to peak voltage of the input and output waveforms. Does the output amplitude agree with what you predicted above?

I claimed that this would form a decent integrator. Let's test that.

Q. Take pictures or sketch the output for sine, square, and triangle waves.

Q. What is the integral of a sine wave, a square wave, and a triangle wave? (Think about what functions describe the parts of the later two). Do your outputs make sense as the integral of the input? (The integral of a triangle wave is NOT a sine.)

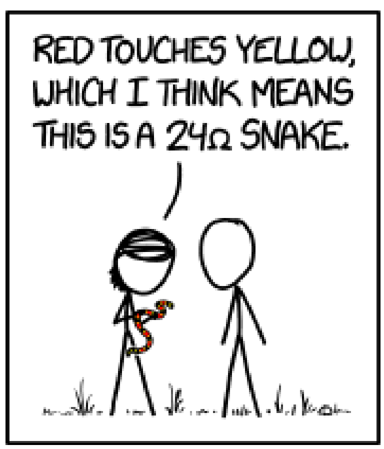

Figure 3: https://xkcd.com/1604/

Part III: Resistor Voltage

Now that we've looked at the voltage across the capacitor in an RC circuit, we are going to examine the voltage across the resistor.

For part III, use the equation for voltage you derived in the prelab.

3.1 RC circuit as a differentiator

We once again want a phase angle near 90 degrees, but this time it will be near positive 90 degrees.

Q. As a function of $f_0 = 1/(2\pi RC)$, what frequency will give a phase shift of +89 degrees? To get closer to 90°, would the frequency need to go higher or lower than that value?

Q. At this frequency, what will the amplitude of the output be (as a fraction of V_{in})?

- Set the function generator to this frequency and follow a procedure similar to that in part II.

Q. Determine the experimental phase angle and output voltage peak to peak amplitude. Compare your results to your predictions.

Q. How does the output amplitude depend on frequency?

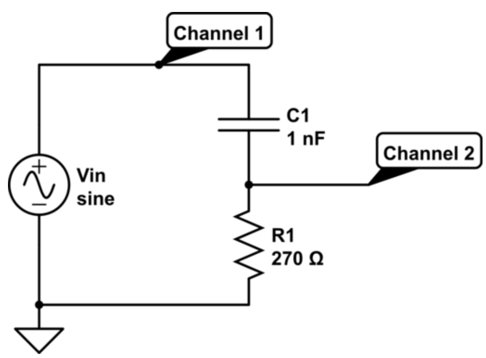

Figure 4: An RC circuit which outputs the derivative of the input.

Q. Examine the output of each wave type at this frequency. Take a picture of, or sketch, each output.

Q. What is the derivative of a sine wave, a triangle wave, and a square wave? Do your outputs make sense as the derivative of the input waveform? Explain.